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DUOPOLY WITH TIME-CONSUMING PRODUCTION

Garth Saloner*

July 1983
(Revised May 1984)

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1. INTRODUCTION

In static oligopoly models in which output is the strategic variable, it is typically assumed that the firms' output choices are made at an instant in time. This is equivalent to assuming either that production is instantaneous or that a firm is committed to producing its entire chosen output. Furthermore, although the order in which the firms make these decisions will usually have a large impact on the outcome (compare the Cournot (1838) and Stackleberg (1934) outcomes), the order is usually exogenously specified. This paper presents a model in which production is time-consuming and where firms can modify their production plans at various times during the production period. Although the production of all firms is assumed to take place contemporaneously, one firm may achieve a *de facto* first-mover advantage. In addition, "degrees" of first-mover advantage emerge endogenously.

The model is essentially a Cournot model in structure with the exception that the single output choice is replaced by a sequence of output choices. Thus this is basically a one-period game with markets clearing only once. Only one aspect of the dynamics of these markets is studied here, namely the time consumed in production. The fact that markets clear only once provides the firms with a commitment to sell everything they produce since unsold goods cannot be carried to a later period. (An explicit multiperiod model with inventory is presented in Saloner (1983)). This considerably simplifies the analysis and focuses attention on the production decision.

The methodology adopted in this model corresponds closely to that developed in Spence (1979) and Fudenberg and Tirole (1981) with two exceptions: (i) the focus here is on production rather than investment, and (ii) the model presented here employs a discrete-time formulation. In the spirit of those models we will usually assume that the amount any firm can produce in a production "sub-period" is bounded above. Consequently it is the relative production rates of the firms that are important in determining the outcome.

In the case where the per-period production is unbounded, even though the firms are competing in outputs and make their output choices simultaneously, and even where all the output is in fact carried out in the first production sub-period, any outcome on the outer envelope of the best-response functions between the Stackelberg outcomes is sustainable as a Perfect Nash Equilibrium. This is in contrast to the usual Cournot outcome.

In the continuous time limit of the discrete time model studied here there is a unique perfect equilibrium path for production. This is in contrast to the multiplicity of equilibria that arise in the Fudenberg and Tirole model. The uniqueness derives from the finite time-horizon rather than the discrete-time formulation. The infinite-horizon setting of the Fudenberg and Tirole model admits implausible perfect equilibria in which each firm invests forever. Given that a firm's opponent is investing, to continue to invest as well is an optimal response. While equilibria of this kind are implausible and uninteresting in themselves, since they are perfect equilibria they can be invoked as perfect equilibrium strategies off-the-equilibrium-path in other equilibria, i.e. the threat to invest forever if an opponent deviates from some specified strategy is credible. These threats can thus be used to sustain other equilibria. The finite-horizon employed here, in contrast, does not admit threats of that kind and hence eliminates equilibria sustained by them. The result is a unique equilibrium for each set of values of the firms' production rates.

We show that a firm with a relatively high rate of production has a degree of first-mover advantage over its opponent. Indeed if the relative rates of production become arbitrarily large, the firm with the higher production rate gains an absolute first-mover advantage as in the Stackelberg model. On the other hand, if the firms are symmetric both in costs and speed of production, the Cournot outcome emerges. The outcome varies continuously as the relative production rate varies between these extremes. Thus the degree of first-mover advantage and hence the appropriate oligopoly solution are endogenously determined and are quite simply related to the production capacities of the firms.

The final result of the model is that unlike the standard static models, the notion of the firm's "size" (as measured by its maximum production rate) takes on some meaning. "Larger" firms in terms of production rates, <u>ceteris paribus</u>, also account for larger shares of total output. Thus if two firms merge, since their combined production rate is at least the sum of their individual production rates, the merged firm will be larger in a meaningful sense. Consequently it is quite simple to generate examples in which firms find it profitable to undertake mergers that result in a decrease in aggregate consumer surplus. This is in contrast to mergers in the Cournot model where a merger essentially results in the disappearance of one of the merging firms.

2. THE MODEL WITH TIME-CONSUMING PRODUCTION

This paper is concerned with providing a means of characterizing a timing of moves that neither endows one of the firms with an absolute first-mover advantage in production (as does the Stackelberg model) nor robs both firms of any first-mover advantage (as does the Cournot-Nash model). In order to focus on the production timing issue, this paper analyzes a single-period model.

The model has the same basic structure as the standard static quantity-setting models. There are two major differences. Firstly, the output and sales decisions of the firms are made separately.[1] Secondly, production is assumed to take time. Formally, the production period consists of T sub-periods with production being allowed each sub-period. After the production period is over the firms then make their decisions about how much to sell. Thus each firm chooses a sequence $\{q_1^i, q_2^i, \ldots, q_T^i, x^i\}$ where q_j^i is the output of firm i in sub-period j and x^i is the amount put up for sale by firm i. The firms make each sub-period choice simultaneously knowing the entire history of all firms' choices to that stage.

We assume that there is some upper bound on the production rate, Q^i, for each firm such that $q_s^i \leq Q^i \forall i$ and s, and we assume constant returns to scale up to Q^i.[2] Our intention is not to impose capacity constraints but rather differential rates of production. Accordingly we will assume that TQ^i is "large" for all i. (If we define the Stackelberg outcome with Firm i as leader as $S^i = (S_1^i, S_2^i)$, for our purposes capacity constraints will not be binding if $(T-1)Q^i > S^i$ for i=1 and 2). It is important to note that even in the special case where Q^i is unbounded, so that both firms could produce their standard cournot outcomes, as will be apparent shortly, the Cournot outcome is not the unique Perfect Nash Equilibrium in this model becasue of the separation of the production and sales decisions. We will ignore the complicating factors of inventory carrying costs and discounting during the production period, both in order to focus on the speed of production and for simplicity.

The standard static best-response functions serve as useful benchmarks. We refer to these as R^1 and R^2 and we denote the outer envelope of R^1 and R^2 by R. The equilibrium strategies will result in discrete-jump production paths moving out from the origin in a northeasterly direction. We will refer to such a path as a production growth path (PGP). We are interested in the properties of equilibrium PGP's. Let $\bar{q}_t^i = \sum_{s=1}^{t-1} q_s^i$ (for t 1,....,T), i.e. $(\bar{q}_t^1, \bar{q}_t^2)$ is the point in output space that the PGP has reached at time t.

Several benchmark regions will prove useful in what follows. These are illustrated in Figure 1 along with a typical PGP.

-- FIGURE 1 --

Regions A,B, & C all lie within R. The borders of the regions are defined by the Nash and Stackelberg points.

The equilibrium concept used is that of a pure strategy Perfect Nash equilibrium. Equilibrium strategies will be denoted by superscripting variables with a star. Where it is important to include the border of a region within that region, the closure will be denoted by a bar.

<u>Lemma 1:</u> If any equilibrium PGP reaches region D, both firms cease production (i.e. if $\bar{q}_s^{1*}, \bar{q}_s^{2*} \varepsilon \bar{D}^*$, $q_t^{i*} = 0 \ \forall t > s, i=1,2$).

<u>Proof:</u> Without loss of generality, suppose that region D is reached on sub-period T-n. Consider sub-period T and suppose that $(\bar{q}_T^{1*}, \bar{q}_T^{2*}) \varepsilon \bar{D}$. Since both firms are on or beyond their best-response functions it is a <u>dominant</u> strategy for each to set $q_T^{i*} = 0$. Now consider sub-period T-1 and suppose that $(\bar{q}_{T-1}^{1*}, \bar{q}_{T-1}^{2*}) \varepsilon \bar{D}$. Regardless of the move made in sub-period T-1 at sub-period T we will have $(\bar{q}^{1*}, \bar{q}^{2*}) \varepsilon \bar{D}$. Therefore neither firm will produce in sub-period T. Therefore the choices at time T-1 will not affect the choices at time T. But then again $q_{T-1}^{1*}, q_{T-1}^{2*} = 0$ are dominant strategies. Clearly this reasoning extends to any sub-period from T-n on. Q.E.D.

<u>Lemma 2:</u> No equilibrium PGP ends in the interior of region D (i.e. $(q_s^{1*}, q_s^{2*}) \varepsilon$

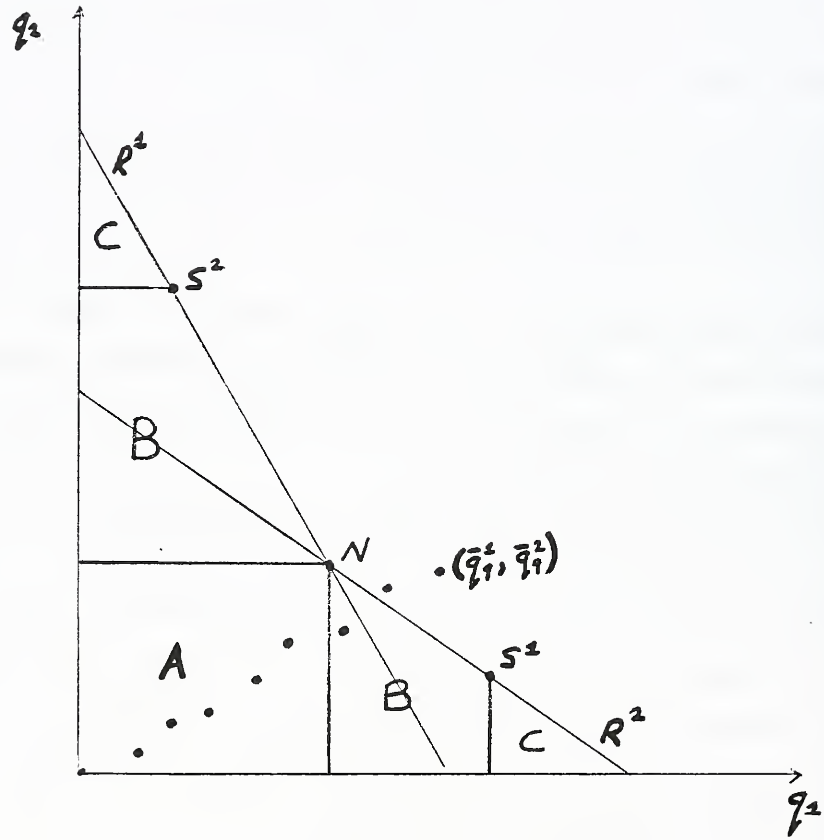

FIGURE 1

interior of D for any s).

Proof: Suppose that a PGP ends in the interior of region D. Let sub-period s be the last sub-period in which there was a strictly positive output by either firm. Then it must be the case that $(\bar{q}_s^1, \bar{q}_s^2) \varepsilon \bar{D}$ by Lemma 1. But then at best one of the folowing must have occured:

a) $q_s^i < R^i(\bar{q}_s^j) - \bar{q}_{s-1}^i$ for i or j, or (b) $\bar{q}_{s-1}^i > R^i(q_s^i)$ and $q_s^i > 0$.

Statement (a) merely says that at least one firm produced an output that took it 'beyond' its best-response function. Statement (b) says that a firm already 'beyond' its best-response function produced a positive output. In case (a) this strategy is dominated by producing $q_s^i = R^i(\bar{q}_s^j) - \bar{q}_{s-1}^i$, and in case (b) by setting $q_s^i = 0$. Q.E.D.

The argument behind Lemma 1 is straightforward but it does display the essential difference between the finite-horizon and infinite-horizon versions of the model. In the infinite-horizon version there are perfect equilibrium paths in which both firms produce as rapidly as possible, even beyond both of their best-response functions. Not only does this mean that there can exist equilibria with terminal points in region D, but 'early-stopping' equilibria in which the firms stop production before the outer envelope of their best-response functions is reached can also be sustained. In these early-stopping equilibria both firms threaten to continue production into region D if both firms do not stop at the target early-stopping point. Since there do exist perfect equilibria that have paths into region D, these threats constitute credible off-the-equilibrium path strategies and indeed are mutually self-enforcing. If one of the firms deviates by producing beyond the target point, it will then be rational for it to participate in its own 'punishment'. In the finite-horizon model, by contrast, there are no equilibrium paths that lead into region D and hence threats to carry production into that region are not credible. As Lemma 3 shows, a major consequence of this is that the 'early-stopping' equilibria disappear.

<u>Lemma 3:</u> Any equilibrium PGP ends in $\bar{D}$ (i.e. $(\bar{q}_T^{1*} + q_T^{1*}, \bar{q}_T^{2*} + q_T^{2*}) \varepsilon \bar{D}$).

<u>Proof:</u> Suppose to the contrary that there is an equilibrium PGP that ends in the interior of A,B or C. Suppose firstly that it ends within the inner envelope of R^1 and R^2. Consider the last sub-period s for which $q_s^{i*} < Q^i$ for at least one i. (One such sub-period exists since TQ^i has been assumed to be sufficiently large.) This strategy for i is dominated by replacing q_s^{i*} in its sequence of moves by $q_s^i = \min\{Q^i, q_s^{i*} + R_s^i(\bar{q}_T^{j*} + q_T^{j*}) - (\bar{q}_T^{i*} + q_T^i).\}$. (This simply says that player i can move the outcome in the direction of his best-response function.)

Suppose then that the PGP ends in the interior of B or C. This means that the PGP crosses exactly one player's best-response function. Without loss of generality let this be player 1. Consider the sub-period T. Regardless of q_T^{i*}, it was a dominant strategy for player 2 to set $q_T^{2*} = Q^2$. Given that it was a dominant strategy for player 2 to set $q_T^{2*} = Q^2$, player 1 must have set $q_T^{1*} = 0$ (for otherwise a reduction by player 1 would have resulted in an outcome nearer his reaction function along the locus $q_2 = \bar{q}_T^{2*} + Q_2$). Consider sub-period T-1 and suppose that $(\bar{q}_T^{1*}, \bar{q}_T^{2*}) \varepsilon$ B or C. Then it must have been the case that $q_{T-1}^{2*} = Q^2$. By the argument for period T player 1 will set $q_T^{1*} = 0$ if region B or C has been reached. But then for any choice of q_{T-1}^{1*} it is a dominant strategy for player 2 to choose $q_{T-1}^{2*} = Q^2$ since the outcome $q_2 = (\bar{q}_{T-1}^{1*} + q_{T-1}^{1*} + 0, \bar{q}_{T-1}^{2*} + 2Q^2)$ is preferable to the outcome $q_2 = (\bar{q}_{T-1}^{1*} + q_{T-1}^{1*} + 0, \bar{q}_{T-1}^{2*} + q_{T-1}^2 + q_T^2)$ $\forall\, q_{T-1}^2, q_T^2 <$

Q^2. Extending this argument backwards in time yields the conclusion that the first step into region B or C and all succeeding steps must have had $q^{1*} = 0$ and $q^{2*} = Q^2$. Now consider the last sub-period s for which $q_s^{1*} \neq 0$ and/or $q_s^{2*} \neq Q^2$ (such a sub-period exists for if $q_t^{1*} = 0$ $\forall t$, the PGP could not cross firm 1's best-response

function). If $q_s^{1*} \neq 0$ then firm 1 could do better by reducing q_s^{1*} and if $q_s^{2*} < Q^2$ firm 2 could do better by increasing q_s^{2*}.

The final possibility to consider is that the equilibrium PGP ends <u>on</u> the inner envelope of the best-response functions at a point other than N. Suppose that there does exist such an equilibrium PGP and call the terminal point A as in Figure 2.

-- FIGURE 2 --

Let sub-period s be the last sub-period for which $q_s^{2*} < Q^2$. Let B $\equiv$ $(\bar{q}_s^{1*} + q_s^{1*}, \bar{q}_s^{2*} + q_s^{2*})$ and let K be the point B shifted vertically by ε. At point B change player 2's strategy for the rest of the game to $q_s^2 = q_s^{2*} + \varepsilon$, $q_t^2 = \min \{Q^2, R_s^2 (\bar{q}_t^{1*}) - \bar{q}_t^{2*}\}$ $\forall t > s$. Given this strategy of player 2, player 1 can guarantee the outcome given by C in the figure by producing a total of $(L - q_s^{1*})$ in the remaining periods. It remains to show that player 1 can do no better than C for if this is the case point C will result from 2's suggested deviation, an outcome that is strictly preferred by player 2. Given player 2's suggested strategy, the outcome will lie on the locus HIJ. All the outcomes on HI with the exception of C are dominated by C itself. Suppose then that player 1 can force an outcome on IJ that is preferred by player 1 to C i.e. that lies on IJ to the southwest of D. Note however that starting from B even by producing as rapidly as possible player 1 cannot

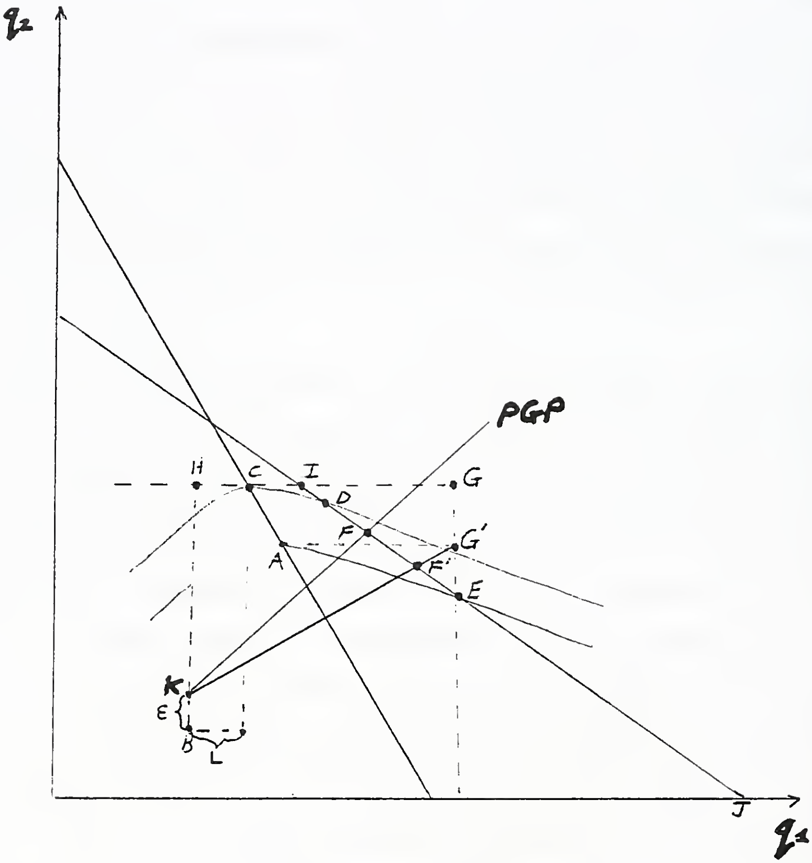

FIGURE 2

attain an output of E_1 before player 2 attains an output of E_2. This follows since if this was possible, adopting that strategy at B would have resulted in a superior outcome for player 1. Also, starting from K, player 2 can attain a total output of exactly C_2. Therefore when player 2 adopts the suggested strategy and player 1 produces as rapidly as possible, the PGP lies on a dotted line originating at K and passing between $H(\varepsilon)$ and $G(\varepsilon)$. Call the point at which this dotted line intersects R^2, F. The closest to J along R^2 that F can lie results when this dotted line passes through G when $\varepsilon=0$ when $G(0)\equiv G'$. Note that this value of F_1' is strictly less than E_1 by the strict concavity of player 1's profit function. Thus facing player 2's suggested strategy, the best outcome that player 1 can attain is F_1', strictly less than E_1. By choosing $\varepsilon>0$ small enough, player 2 can ensure that $F_1^i <$ D_1 and hence that C is preferred by player 1 to F. Thus player 2's deviation results in point C which with $\varepsilon>0$ is strictly preferred by player 2 to the original outcome A. Q.E.D.

Lemmas 1, 2, and 3, taken together, imply that any equilibrium PGP, if one exists, must terminate on R. In fact an equilbrium does exist and in general there will be a continuum of equilibria. However, in the limit, as the length of each period goes to zero, we approach a unique equilibrium which is the analog of the equilibrium in Spence's model (1979).

Denote the game described above by Γ. Now consider any subgame in which it is possible for the firms to reach R in a single move or in which either of the firms can attain its Stackelberg output in a single move. (If only one firm can attain its Stackelberg output let it be Firm 1). Further suppose there are sufficiently many sub-periods remaining for both firms to be able to reach their own best-response functions. Denote such a subgame by Γ_s. Let the portion of R between S^1 and S^2 inclusive be denoted by F. Finally let the cumulative outputs of the two firms at the start of this subgame be (a_1, a_2) and define $E = F \cap \{[a_1, a_1 + Q_1] \times [a_2, a_2 + Q_2]\}$.

Proposition 1: If E is nonempty the elements of E are the Perfect Nash Equilibria of Γ_s. If E is empty, the unique Perfect Nash Equilibrium is S^1.

Proof: If E is empty Firm i (w.l.o.g. let this be Firm 1) can attain its Stackelberg output before is opponent can reach its own best-response function. Label the sub-periods of Γ_s by s1, s2, ... Let the output of Firm 1 in sk be q^1_{sk}. The following strategies constitute a Perfect Nash Equilibrium: Firm 1 produces $S_1 - a_1$, and Firm 2 produces Q^2 in s1. In later sub-periods, ℓ, Firm 2 produces $\min \{Q^2, R^2_s(a_1 + \sum_{k=1}^{\ell-1} q^1_{sk})\}$ i.e. Firm 2 attempts to produce its best-response to Firm 1's cumulative output at the end of the previous sub-period. Since (1) any equilibrium must lie on R by Lemmas 1-3, (ii) Firm 1 can guarantee the outcome S^1 by following the suggested strategy, and (iii) S^1 is Firm 1's most preferred point on R, S^1 is the unique Perfect Nash Equilibrium.

Now suppose that E is nonempty. Let $(b_1, b_2) \in E$. The outcome (b_1, b_2) lies on R. Without loss of generality suppose it lies on R^1. The following strategies constitute a Perfect Nash Equilibrium: In sub-period s1 Firm 1 Produces $b_1 - a_1$ and Firm 2 produces $b_2 - a_2$. In any later sub-period, ℓ, Firm i produces

$$\min \{Q^i, R^i(a_j + \sum_{k=1}^{\ell-1} q^j_{sk})\}.$$

These strategies result in the outcome (b_1, b_2) in equilibrium. Clearly no firm has an incentive to deviate. Given its opponent's strategy, increasing output in the first sub-period of Γ_s results in an outcome in the region D or on the portion of the outer envelope that borders region C both of which are strictly worse than the outcome on the equilibrium path. If a firm decreases its output in that sub-period, its opponent produces a positive amount the following sub-period. Since the strategies lead to an outcome that at least reaches the outer envelope of the best-response functions the result is strictly worse for the deviator. Since (b_1, b_2) is on the outer envelope any deviation in a sub-period after s1 makes both firms strictly worse off. This establishes that any element of E is sustainable as the outcome of a Perfect Nash Equilibrium. It remains to show that there are no equilibria that are not in E. Let $(\overline{b}_1, \overline{b}_2)$ be the element of E that is least preferred by Firm 1. Firm 1 can guarantee at least this outcome by producing $\overline{b}_1 - a_1$ in sub-period s1. Similarly for Firm 2. Q.E.D.

Note that in the special case in which Q^1 and Q^2 are unbounded, any outcome between S^1 and S^2 inclusive is sustainable as a Perfect Nash Equilibrium. Merely adding multiple sub-periods of production to the standard static Cournot model eliminates the uniqueness of the Cournot equilibrium and vastly enlarges the set of possible equilibria, which then includes the Stackelberg outcomes!

Proposition 2: The following strategies form Perfect Nash Equilibria for Γ: Let Γ_t be the subgame starting at time t. If Γ_t satisfies the requirements of Γ_s, follow strategies in Proposition 1. Otherwise set $q^i_t = Q^i$ regardless of the history of play to date.

In Γ, firms produce as rapidly as possible until they reach a subgame described by Γ_s. They then follow equilibrium strategies for that subgame. It is easily verified that these are equilibrium strategies and a formal proof is omitted. Notice that since Γ_s will in general have multiple equilibria, so will Γ. Furthermore, the size of the set of equilibria will depend on how close to R the point (a_1, a_2) is. In particular, as $(a_1, a_2) \to R$ the set of equilibria approaches (a_1, a_2) as the unique equilibrium. This motivates the following:

Proposition 3: Let Q^1, $Q^2 \to 0$ holding Q^1/Q^2 constant. Then there is a unique equilibrium to Γ. If the PGP intersects F the equilibrium is the point of intersection on F. Otherwise if R^i is crossed by the PGP before R^j, the unique equilibrium outcome is S^i.

Proof: Consider the case where the PGP intersects F. Consider the equilibrium strategies described in Proposition 2. Let $(a_1(Q_1), a_2(Q_2))$ be the starting point of Γ_s. Let $(\bar{a}_1, \bar{a}_2)$ be the point of intersection of the PGP and F when the PGP and F when the PGP is continuous. As $Q_1, Q_2 \to 0$, $\{(a_1, a_1 + Q_1), (a_2, a_2 + Q_2)\} \to \{(\bar{a}_1, \bar{a}_2)\}$ and $F \cap \{(a_1, a_1 + Q_1), (a_2, a_2 + Q_2)\} \to (\bar{a}_1, \bar{a}_2)$. This establishes that $(\bar{a}_1, \bar{a}_2)$ is an equilibrium as $Q_1, Q_2 \to 0$. As before, the fact that (i) any equilibrium PGP ends on R, (ii) firm i can guarantee a terminal output of at least $\bar{a}_i$, and (iii) the firms have strictly opposing preferences along R, imply that $(\bar{a}_1, \bar{a}_2)$ is the unique equilibrium. The argument for S^i is similar. Q.E.D.

In the continuous time version of this model the relative rates of production determine the unique outcome. If the production rate of one of the firms is sufficiently large relative to its opponent, it will be able to achieve its Stackelberg outcome and it will effectively have a first-mover advantage. As the ratio Q_2/Q_1 rises, the equilibrium outcome changes continuously from S^1 to S^2. The Cournot-Nash outcome has no special significance and emerges only when the relative rate of production of Firm 1 to Firm 2 is in the ratio N_1/N_2.

As was mentioned in the introduction, the fact that the outcome depends on the relative rates of production means that the concept of 'size' takes on some meaning here. Suppose that when two firms merge that the production rate of the merged entity is the sum of the individual rates. In this case the new firm will be 'larger' than the merging firms and may attain a more dominant position in the industry.

As an example consider an industry of three firms where the rates of production are in the ratio $Q^1:Q^2:Q^3 = 2:1:1$. Let all the firms have constant marginal costs (normalized to be zero) and suppose the industry inverse demand function is $P = 15-q$, where $q = q_1+q_2+q_3$ is the total industry output at the end of the period. The outcome in this model has $q_1 = 6$, $q_2 = q_3 = 3$, $P = 3$, consumer surplus of 72 and profits of 18, 9 and 9 for Firms 1, 2 and 3 respectively. Suppose now that Firms 2 and 3 merge to form Firm 0. In this case the rates of production of Firms 0 and 1 are equal. The equilibrium outcome has $q_0 = q_1 = 5$, $P = 5$, consumer surplus of 50 and profits of 25 for each firm. Notice that Firm 0 earns more than the sum of the pre-merger profits of Firms 2 and 3. Thus an incentive to merge exists and the merger leads to a decrease in both consumer surplus and social welfare (as measured by the sum of consumer and producer surplus).

In this example the merger both destroys Firm 1's leadership and reduces the competitiveness of the industry. It is the combination of both of these effects that leads to the decrease in social welfare. To illustrate this, suppose instead that Firms 1 and 2 merge to form a new Firm 0. In equilibrium $q_0 = 9$ and $q_3 = 3$. Thus the output of the merged firms is merely the sum of the pre-merger firms' outputs. Accordingly prices and welfare are unchanged. The reason for this is straightforward. In equilibrium Firm 3's output will be one-fourth of the total, both pre- and post-merger. Thus from Firm 3's perspective the PGP remains the same. Further, in both cases Firm 3 is a "follower". Thus the equilibrium, which is at the intersection of the PGP and Firm 3's best-response function, remains unaltered.

Thus the example is not meant to illustrate that an incentive to undertake welfare-reducing mergers always exists, but rather that a key ingredient to a theory of mergers is a notion of firm size that is preserved (at least to some degree) when mergers take place. In this model this is achieved by merging the firms' production rates. More generally, merged firms should also be expected to preserve existing customer relationships and brand loyalties.

3. CONCLUSION.

Traditional models of oligopoly usually do not embody any notion of firm size. A result of this is the fact that firms that have equivalent cost structures are placed in a symmetric position with respect to the shares of final output that they command regardless of their capacities. Thus when two firms merge the result is simply that one decision-making agent who was previously active disappears. Therefore in a Cournot model with more than two symmetric firms, a merger by any two firms results in a decrease in profits to the merging firms. There are, however, a number of reasons why firm size may be important. The reason explored in this paper relates to the relative speeds of production of the competing firms in a model in which production is time-consuming. In such a setting large firms, i.e., those with high maximum production rates, are able to gain a degree of first-mover advantage. Other reasons for the importance of firm size include customer loyalties that are not entirely lost when firms merge and the reputations that the firms' products themselves enjoy. Any of these factors will be sufficient for profitable mergers that are socially detrimental to be possible.

It is not unusual for the equilibria in games with finite time horizons to differ from those in their infinite-horizon counterparts. This has been shown to be the case here and the factors leading to the difference have been identified. However the reduction of the multiplicity of equilibria to the unique equilibrium found here as the continuous time limit of the discrete finite game is only of interest if one is prepared to rule out the perfect equilibrium strategies that give rise to the mutiplicity or if the finite time-horizon intrinsically has more appeal. In product markets where there are frequent model changes, either by the firms' choice or due to obsolescence, the appropriate model has a finite time-horizon and thus also a unique equilibrium.

Finally, the model presented here provides a means of endogenizing the degree of first-mover advantage a firm possesses in a quantity-setting oligopoly model. A firm's degree of leadership or followership is linked here to its production capacity. This raises a number of issues for entry deterrence and strategic choice of capacity that have not been addressed here.

Notes

1. This follows the methodology used in Saloner (1983).

2. This assumption follows the Fudenberg and Tirole (1981) model.

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